Relative equilibria of the curved N-body problem

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This monograph provides a mathematical proof that, for distances of the order of 10 AU, space is Euclidean. This result is, of course, not surprising for such small cosmic scales. But it is good to finally have a mathematical confirmation in this sense. This book shed some light on the dynamics of N point masses that move in spaces of non-zero constant curvature according to an attraction law that naturally extends classical Newtonian gravitation beyond the flat (Euclidean) space. This extension is given by the cotangent potential, proposed by the German mathematician Ernest Schering in 1870. He was the first to obtain this analytic expression of a law suggested decades earlier for a 2-body problem in hyperbolic space by Janos Bolyai and, independently, by Nikolai Lobachevsky. As Newton's idea of gravitation was to introduce a force inversely proportional to the area of a sphere the same radius as the Euclidean distance between the bodies, Bolyai and Lobachevsky thought of a similar definition using the hyperbolic distance in hyperbolic space. The recent generalization we gave to the cotangent potential to any number N of bodies, led to the discovery of some interesting properties. This new research reveals certain connections among at least five branches of mathematics, classical dynamics, non-Euclidean geometry, geometric topology, Lie groups, and the theory of polytopes.